Nearest Neighbors in 2D Statistical Geometry Fractals.

1. Neighbors -- Circle Case.

Figure 1 shows a typical example of a statistical geometry fractal, in which circles are fractalized within a circular boundary.

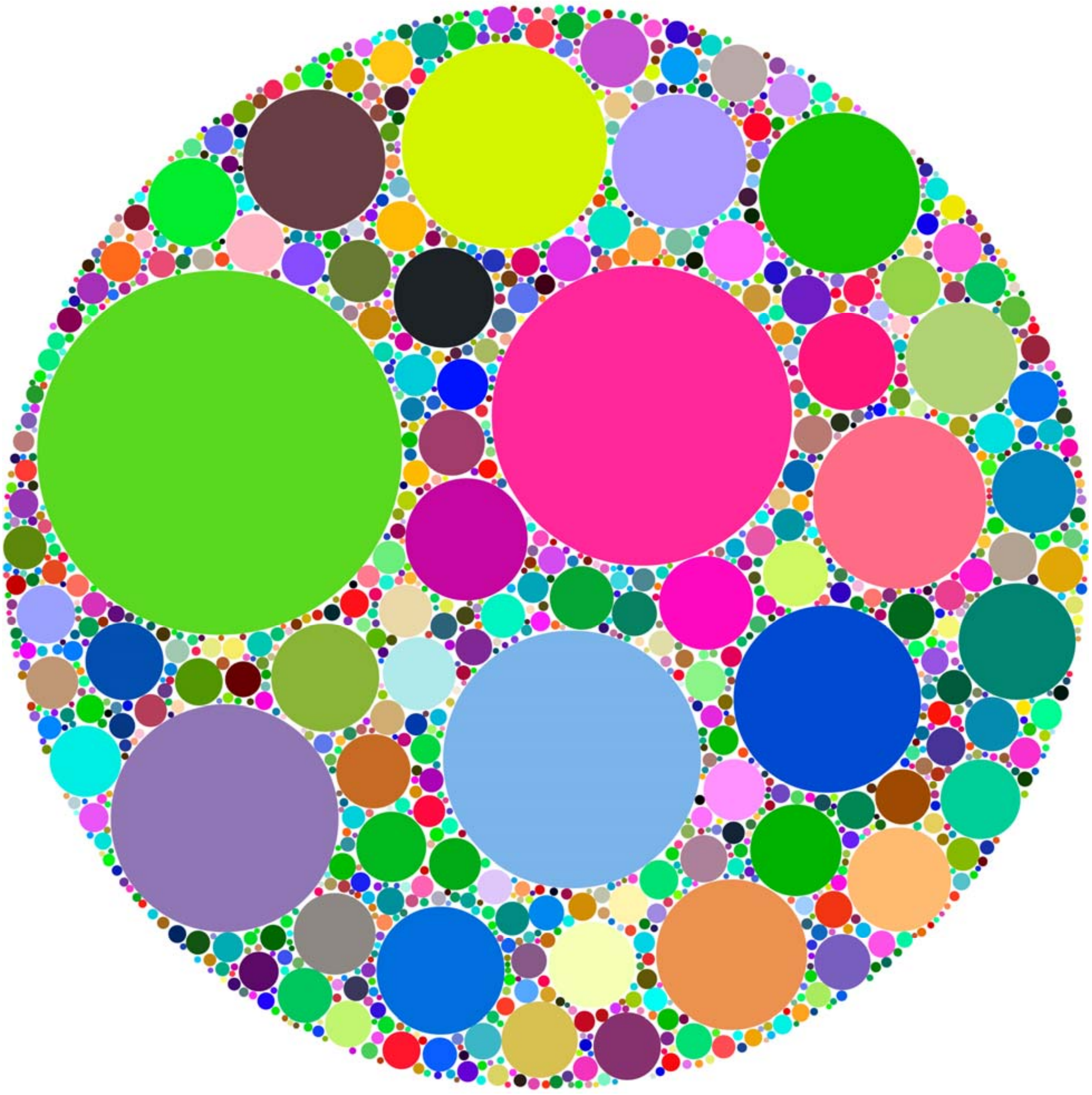

Fig. 1 Circle fractal with c=1.36, N=3, 1500 circles. Random color.

In space-filling periodic tilings the nearest neighbors repeat and are easily enumerated. In a space-filling fractal of the kind considered here it is evident by inspection that each shape has some near neighbors, but exactly how one identifies them is problematic. If we view the fractal as carried out "to infinity" the answer must be that each shape has an infinite number of nearest neighbors, which while it may be true isn't very informative.

We will consider fractals with a finite number of shapes. It has been found [6] that the circle-circle edge-to-edge distances have a distribution of values, down to zero.

2. Neighbors in a Proximity Tree; Circles

A *proximity tree* is here defined as a graph in which any two shapes whose mutual spacing (edge-to-edge) is less than some given value have a line drawn between their two centers. The lines thus denote neighbors. It is obvious that the results will depend on what value we choose for this maximum spacing.

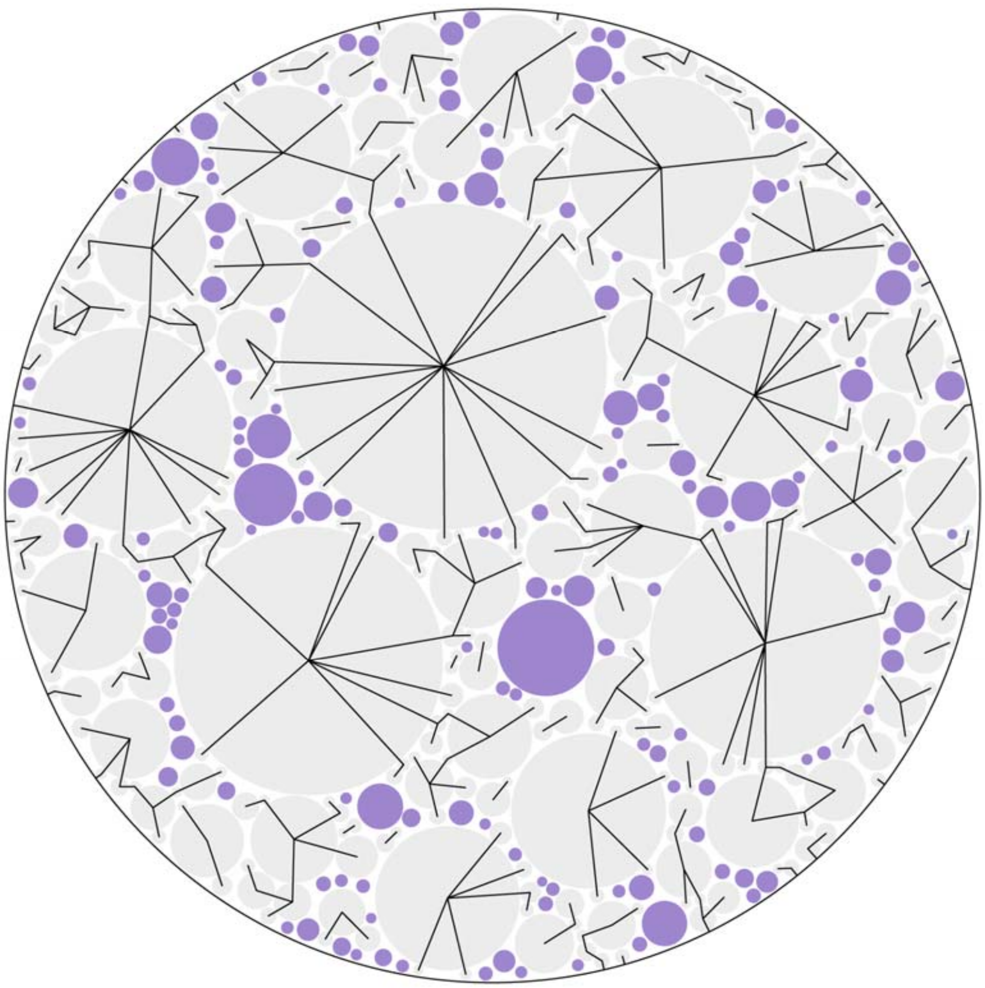

Fig. 2. Circle fractal with c=1.36, N=3, 500 circles. Random color. r_{max}=.16$r_{smallest}$ where r_{max} is the largest spacing, and $r_{smallest}$ is the radius of the smallest placed circle. The non-connected circles are shown in purple. The outer bounding circle is, in effect, yet another circle and those circles meeting the proximity test with respect to the outer boundary are shown linked to it.

According to previous studies [6] the average edge-to-edge spacing will scale down with increasing number of shapes in proportion to the radius of the smallest (last) shape, so it is reasonable to scale the maximum spacing with this radius.

In all cases considered here the fractals are "space-filling in the limit". The definitions of c, N, etc. can be found in various reports by the author [2-8].

The number of near neighbors can be found by counting the number of lines radiating from the center of a given circle. It will be a random quantity, although on average we would expect it to grow in proportion to the circumference of the given circle, and this is qualitatively seen in the image.

This graph is random and looks random, but it should be kept in mind that the randomness in these fractals is highly constrained. This graph has no "crossovers", i.e., it is a planar graph. It also has rings, although they are not as common as one might expect from pure randomness.

The appearance of such a tree varies greatly as a function of the ratio $r_{max}/r_{smallest}$. If this quantity is large the tree will be nearly a single connected whole, with only a few small disconnected nets. If it is small the tree will be composed of a large number of small disconnected nets. The case shown is intermediate between these limiting cases.

When there are numerous disconnected nets the number of branches and nodes in the nets appear to follow an approximately fractal trend, with a few big nets and many small ones. The unconnected (purple) circles can be viewed as nets with 1 node and zero branches.

The nets appearing in these graphs do not appear to have the "small world" property, i.e., one cannot go from any node to any other in an ensured small number of jumps, i.e., they don't have "long range" branches. This is assumed to arise from the geometric effects of the 2D constraint.

The computation method is to compute mutual spacings for all circle pairs.

A large effort could be expended in studying the statistical properties and distributions of these trees and the nets they are composed of, since there are many parameters -- total number of shapes, c, N, ratio $r_{max}/r_{smallest}$, etc.

Some viewers have seen a resemblance to the constellation charts of astronomy. Others see a maze to be traced. The latter view is enhanced if the image is drawn as lines-only, without circles (see Fig. 4).

3. Neighbors in a Proximity Tree; Squares.

Figure 3 shows the corresponding tree for squares. It looks qualitatively much the same as the circle case.

Mutual distance requires a careful definition for squares. In this instance the "Manhattan distance" has been computed. If another shape's corner falls within the x-range or y-range of the given shape, the corresponding rectilinear distance is computed. If the shortest distance is at an angle from corner to corner they are viewed as non-neighbors.

The connections to the bounding square are not shown.

Fig. 3. Square fractal with c=1.36, N=3, 500 circles. Random color. d_{max}=.24$d_{smallest}$ where d_{max} is the largest spacing, and $d_{smallest}$ is the side half-length of the smallest placed square. The non-connected squares are shown in purple.

4. Statistics of Inter-Shape Spacings.

This has been studied previously for circles [6]. The histograms show that the probability distribution function (p.d.f.) *p(d)* for short circle-circle distances *d* is well fitted by a decreasing exponential function.

$$p(d) = p_0 e^{-d/d_0}$$

where p_0 and d_0 are parameters which depend on c, N, number of shapes placed, etc. This has not been studied for the squares case.

5. Discussion.

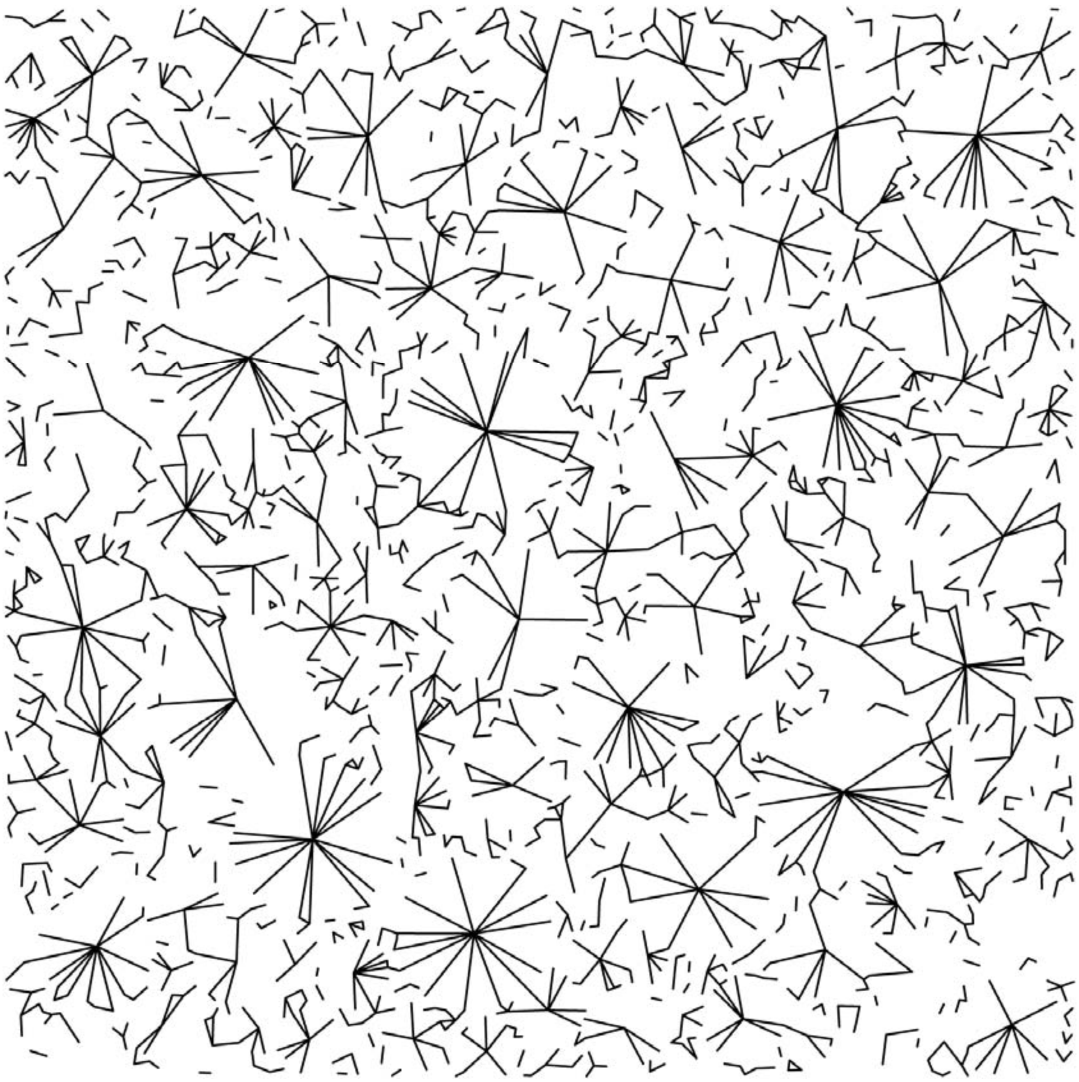

Fig. 4. Circles-in-square proximity tree with c=1.36, N=20, 3000 circles. The circles are not drawn, so that only the tree skeleton is seen. The connected nets making up the tree vary greatly in size.

A new class of random trees (graphs, nets) is described. These trees appear to have many fractal properties, which is not surprising in view of the manner in which they are created.

Are these trees of interest as models? Perhaps. One situation that comes to mind is the spread of species among islands. If the inter-island distance is short enough, the species spreads, while for large distances it fails to spread. The connected nets would then be maps of locations where a given species exists. A related situation is the spread of disease agents (virus, bacterium, parasite) within a population. The exponentially decaying distribution of inter-shape distances is interesting in this connection.

6. References.

[1] "Fractals: Form, Chance and Dimension", Benoit Mandelbrot (1977). The *opus magnus* of fractals.

[2] John Shier, "Hyperseeing", Summer 2011 issue, pp. 131-140, published by ISAMA. Available by download at the web site john-art.com.

[3] "Statistical Geometry", John Shier, July 2011. A colorful self-published fractal art picture book available at lulu.com.

[4] "The Dimensionless Gasket Width *b(c,n)* in Statistical Geometry.", John Shier. Report 1 in a series. Available at the site john-art.com.

[5] "Holes in Circle Fractals. Statistics.", John Shier. Report 2 in a series. Available at the site john-art.com.

[6] "Nearest Neighbor Distances in Circle Fractals. Statistics.", John Shier. Report 3 in a series. Available at the site john-art.com.

[7] "Multishapes and Packability in Statistical Geometry.", John Shier. Report 4 in a series. Available at the site john-art.com.

[8] "Statistics of Trials and Placements: Circles and Squares. The Parameters *c* and *f*.", John Shier. Report 5 in a series. Available at the site john-art.com.